

5-12 Solving Equations by Factoring

Objective: To use factoring in solving polynomial equations.

Vocabulary

- Zero-product property** A product of factors is zero if and only if one or more of the factors is zero.
- Polynomial equation** An equation whose sides are both polynomials.
- Linear equation** A polynomial equation whose term of highest degree has degree 1.
For example, $x - 2 = 0$ and $5x - 4 = 6$.
- Quadratic equation** A polynomial equation whose term of highest degree has degree 2.
For example, $x^2 - x - 6 = 0$, $x^2 = 9x$, and $10x - 9 = x^2$.
- Cubic equation** A polynomial equation whose term of highest degree has degree 3.
For example, $x^3 - 2x^2 + x - 1 = 0$.
- Standard form of a polynomial equation** A form of an equation in which one side is a simplified polynomial arranged in order of decreasing degree of the variable and the other side is zero.
- Double or multiple root** A factor that occurs twice in the factored form of an equation.
For example, 5 is a double root of $x(x - 5)(x - 5) = 0$.

Example 1 Solve $(x - 1)(x + 3) = 0$.

Solution Since the product of factors is 0, one of the factors on the left side must equal 0.

$$x - 1 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 1 \quad \quad \quad x = -3$$

The solution set is $\{1, -3\}$. Just by looking at the original equation, you can see that when $x = 1$ or $x = -3$, the product will be 0.

Example 2 Solve $3n(n - 2)(n - 5) = 0$.

Solution $3n = 0$ or $n - 2 = 0$ or $n - 5 = 0$
 $n = 0$ $n = 2$ $n = 5$ The solution set is $\{0, 2, 5\}$.

CAUTION Never transform an equation by dividing by an expression containing a variable. Notice that in Example 2, the solution 0 would have been lost if both sides of the equation had been divided by $3n$.

- Solve.**
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|----------------------------|-------------------------|----------------------------|--------------------|
| 1. $(y + 4)(y - 5) = 0$ | 2. $0 = (n + 1)(n + 8)$ | 3. $10n(n - 2) = 0$ | {0, 1, 3} |
| 4. $2x(x - 10) = 0$ | 5. $(p - 1)(p - 7) = 0$ | 6. $0 = 2n(n - 1)(n - 3)$ | {0, 10} |
| 7. $x(2x - 1)(2x + 1) = 0$ | 8. $0 = n(n - 6)$ | 9. $0 = 3x(4x - 1)(x - 2)$ | {0, 6} |
| | | | {0, 1/4, 2} |

5-12 Solving Equations by Factoring (continued)

Example 3 Solve the quadratic equation $2x^2 - x = 3$.

Solution

- Transform the equation into standard form. $2x^2 - x - 3 = 0$
- Factor the left side. $(2x - 3)(x + 1) = 0$
- Set each factor equal to 0 and solve. $2x - 3 = 0$ or $x + 1 = 0$
 $2x = 3$ $x = -1$
 $x = \frac{3}{2}$
- Check the solutions in the original equation.

$$2\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) \stackrel{?}{=} 3 \qquad 2(-1)^2 - (-1) \stackrel{?}{=} 3$$

$$2\left(\frac{9}{4}\right) - \frac{3}{2} \stackrel{?}{=} 3 \qquad 2(1) + 1 \stackrel{?}{=} 3$$

$$\frac{9}{2} - \frac{3}{2} \stackrel{?}{=} 3 \qquad 3 = 3 \checkmark$$

$$\frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3 \checkmark$$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

- Solve.**
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|--|--|--|
| 22. $\{-7, -5\}$ | 28. $\left\{\frac{1}{3}, 1\right\}$ | 30. $\left\{8, -\frac{10}{3}\right\}$ |
| 10. $x^2 - x - 12 = 0$ {4, -3} | 11. $x^2 - 12x + 27 = 0$ {3, 9} | 12. $0 = x^2 - 4x - 32$ {8, -4} |
| 13. $0 = m^2 + 3m - 54$ {6, -9} | 14. $x^2 - 4y + 3 = 0$ {1, 3} | 15. $x^2 - 10x - 24 = 0$ {12, -2} |
| 16. $0 = n^2 - n$ {0, 1} | 17. $y^2 = 12y$ {0, 12} | 18. $6k^2 = 2k$ {0, 1/3} |
| 19. $x^2 + 16 = 8x$ {4} | 20. $a^2 = 10 - 3a$ {-5, 2} | 21. $3x^2 - x = 2$ {-2/3, 1} |
| 22. $0 = x^2 + 12x + 35$ | 23. $y^2 + 5y = 14$ {-7, 2} | 24. $x^2 = 5x + 36$ {-4, 9} |
| 25. $4m^2 - 25 = 0$ {-5/2, 5/2} | 26. $r^2 + 8 = 9r$ {1, 8} | 27. $6n^2 - n = 2$ {2/3, -1/2} |
| 28. $3x^2 + 1 = 4x$ | 29. $3a^2 = 6a$ {0, 2} | 30. $3p^2 - 14p = 80$ |
| 31. $2x^2 = 10 + x$ {-2, 5/2} | 32. $3p^2 + 17p = -10$ | 33. $3x^2 + 1 = 4x$ {1/3, 1} |
| | | {-2/3, -5} |

Mixed Review Exercises

- Evaluate if $x = 3$ and $y = 6$.**
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|----------------------------|-------------------------------|-------------------------|
| 1. $(x - y)^3 - 27$ | 2. $x^3 \cdot x^2$ 243 | 3. $4x^3$ 108 |
| 4. $(4x)^3$ 1728 | 5. $3x + y^2$ 45 | 6. $3x^2 + y$ 33 |
| 7. $3(x + y)^2$ 243 | 8. $(yx)^2$ 324 | 9. y^2x^2 324 |
- Simplify.**
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|---|---|--|
| 10. $(5x^2y^2)(-3xy^4)$ $-15x^3y^6$ | 11. $(8a)^3$ $512a^3$ | 12. $-3(x + 4)$ $-3x - 12$ |
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